বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examination 2022

(Under CBCS Pattern)

Semester - VI

Subject : MATHEMATICS

Paper : SEC 4 - T

Full Marks : 40

Time : 2 Hours

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

[PROBABILITY AND STATISTICS]

1. Answer any ***four*** questions : 5×4=20

(a) A point R is chosen at random on a line segment OA of length $2a$. Find the probability that the area of the rectangle with its sides OR and RA will never exceed $\frac{a^2}{2}$. 5

(b) (i) If X is a Poisson (μ) variate, then find the probability distribution of X^3.

(ii) Find the mean and variance of the rectangular distribution. 3+2

(c) (i) Show that correlation coefficient between two random variables is numerically 1, if and only if they are linear.

(ii) The regression lines of two random variables X and Y are $2x+3y=5,\ 5x+2y=8$. Find their correlation coefficient. 2+3

(d) For a normal $(m,\ \sigma)$ distribution, prove that $\mu_{2r}=1.3.5......(2r-1)\sigma^{2r}$, where μ_r represents r-th order central moments and r being a positive integer. 5

(e) If X is a binomial *(n, p)* variate, then prove that $\mu_{k+1}=p(p-1)\left(nk\mu_{k-1}+\frac{d\mu_k}{dp}\right)$,

where μ_k is the k^{th} order central moment. 5

(f) Two discrete random variables X and Y have joint probability mass function given by the following table :

		Y		
		1	2	3
	1	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
X	2	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$
	3	$\frac{1}{12}$	$\frac{1}{12}$	0

Compute the probability of each of the following events :

(i) $X\le 1.5$ (ii) XY even ((iii) Y is even given that X is even. 5

2. Answer any ***two*** questions : 10×2=20

(a) (i) State and prove Baye's theorem.

(ii) Show that the probability that exactly one of the events A and B occurs is $P(A)+P(B)-2P(AB)$.

(iii) If $f(x)=A\ Exp\left[-\frac{1}{2b^2}(x-a)^2\right]$, $-\infty<x<\infty$, find A so that *f(x)* is a probability density function. 4+3+3

(b) If the joint pdf of the random variables X, Y is

$$f_{x,y}(x,y)=\begin{cases}kx(3x+y), & 0\le x\le 3,\ 0\le y\le 2\\ 0, & elsewhere\end{cases}.$$

Find (i) $P(X+Y<2)$ (ii) the marginal distributions of *X* and *Y*, Investigate whether *X* and *Y* are independent. 5+4+1

(c) (i) If *X* is uniformly distributed over [1, 2], find *K* so that $P(X>K+E(X))=\frac{1}{6}$.

(ii) A jar contains three white and three red balls. The balls are drawn at random from the jar and placed on a table in the order drawn. What is the probability that balls are drawn in the order white, red, red, white, red, white? 5+5

(d) (i) A dart is thrown at random on a square target board having vertices (1, 0), (0, 1), (–1, 0) and (0, –1), the point at which the dart hits the board being (*X*, *Y*), find the marginal density functions of *X* and *Y* and show that they are dependent.

(ii) Let X_1, X_2, X_3 be three independent $N(\mu,1)$ variables. Prove that $P[X_1-2x_2+x_3>0]=\frac{1}{2}$. 6+4

OR

[FORECASTING]

1. Answer any ***four*** questions : 5×4=20

   (a) Discuss the overall purpose people have for investing. Define investment.

   (b) Briefly discuss the five fundamental factors that influence the risk premium of an investment.

   (c) On August 15, you purchased 100 shares of stock in a Company X at \$65 a share and a year later you sold it for \$61 a share. During the year, you received dividends of \$3 a share. Compute your HPR and HPY on your investment in X.

   (d) Discuss the process of double exponential smoothing.

   (e) Discuss the key factors of time series in business forecasting.

   (f) What are the disadvantages of forecasting?

2. Answer any ***two*** questions : 10×2=20

   (a) Given the weekly demand data, what are the exponential smoothing forecasts for Periods 2-10 using :

   A) $\alpha = 0.10$, B) $\alpha = 0.60$

   Assume F1 = D1

Week	1	2	3	4	5	6	7	8	9	10
Demand	820	775	680	655	750	802	798	689	775	?

   (b) Given forecast errors of –1, 4, 8 and –3, what is the mean absolute deviation?

   (c) In a simple linear regression model, $y_i = \beta x_i + \varepsilon_i$ where ε_i is $N\left(0, \sigma^2\right), i = 1, 2, \ldots\ldots, n$. Determine an unbiased estimator of σ^2.

   (d) Suppose there are 5 explanatory variables (including an intercept term) in the model $y = X\beta + \varepsilon$ and $n = 26$. Further, the sum of squares due to total is obtained as 300 and sum of squares due to error is obtained as 100. Find the value of F-statistic in the context of analysis of variance.

OR

[PORTFOLIO OPTIMIZATION]

1. Answer any ***four*** questions : 5×4=20

(a) Define (i) Beta of a portfolio (ii) Security market line.

(b) You have a portfolio with a beta of 0.84. What will be the new portfolio beta if you keep 85% of your money in the old portfolio and 14% in a stock with a beta of 1.93?

(c) What are some of the benefits of diversification?

(d) Use the information in the following to answer the questions below :

State of Economy	**Probability of State**	**Return on A in State**	**Return on B in State**
Boom	35%	0.040	0.210
Normal	50%	0.030	0.080
Recession	15%	0.046	–0.010

(i) What is the expected return of each asset?

(ii) What is the variance of each asset?

(e) Define the Sharpe, Trey nor, and Jensen measures of portfolio performance evaluation.

(f) Explain different types of risks in a portfolio optimization.

2. Answer any ***two*** questions : 10×2=20

(a) Explain Sharpe model for determining the efficient portfolio set.

(b) Explain the capital asset pricing model (CAPM).

(c) Explain Markowitz approach to portfolio selection.

(d) Write a note on financial markets.